MULTIMEDIA UNIVERSITY

STUDENT ID

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

DMA5301 – Discrete Structures

(All sections / Groups)

08 FEBRUARY 2023
9.00 a.m - 11.00 a.m
(2 Hours)

QUESTION	MARKS
Q1	
Q2	
Q3	
Q4	
TOTAL	

INSTRUCTIONS TO STUDENTS

1. Write your student ID in the boxes provided on the top right of this cover page.
2. This question paper consists of 9 pages, including the cover page, with FOUR questions only.
3. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. All the answers must be written in the space provided below each question.

Answer ALL Questions

QUESTION 1

(a) Let the universal set $U = \{x \mid x \in Z^+ \wedge x < 14\}$, $A = \{x \mid x \text{ divides } 12\}$ and $B = \{x \mid x \text{ is a prime number}\}$.

i. $U - (A \cup B)$ [1 mark]

ii. $B - A$ [1 mark]

iii. $|A - B|$ [1 mark]

iv. $(A - B) \cap (B - A)$ [1 mark]

Continued .....

(b) Given a relation $R = \{(1,1), (1, 2), (2,1), (2,2), (3,2), (3,3)\}$ on the set $S = \{1, 2, 3, 4\}$.

i. Is R symmetric? Justify your answer. [2 marks]

ii. Is R transitive? Justify your answer. [2 marks]

iii. Is R a function? Justify your answer [2 marks]

Continued .....

QUESTION 2

(a) Find $lcm(67, 985)$. [4 marks]

(b) Find $f(2)$ and $f(3)$ if $f(n)$ is defined recursively by

$$f(0) = 1, f(n+1) = 2f(n) + 3, n \geq 0.$$ [2 marks]

Continued .....

(c) Prove that $f(n) = \frac{n+2n^2}{2}$ is $O(n^2)$. You must show all intermediate steps in your solution. [4 marks]

QUESTION 3

(a) Given the predicate $Q(x, y)$: $x + y = 5$, with the domain of discourse = Z^+. Determine the truth value for each of the following statements. Justify each of your answer.

i. $\exists x Q(x, 3)$ [1 mark]

ii. $\forall x \exists y Q(x, y)$ [2 marks]

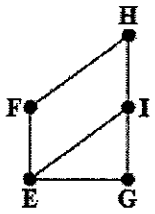
Continued .....

(b) Proof that $(q \wedge r) \rightarrow (r \wedge p)$ is logical equivalent to $\neg r \vee \neg q \vee p$ by using laws of equivalence. You must state the law used in each step clearly. [7 marks]

Continued .....

QUESTION 4

(a) Find a spanning tree for the graph below using the Breadth-first search method. Assume that vertex E is the root and the selection of vertices to the queue must follow alphabetical order.



You must present your answer in the table format shown below and must show all the steps clearly.

Step	Queue	Tree
1	E	-
⋮	⋮	⋮

[4 marks]

Continued .....

(b) Determine if the graph given in Question 4(a) has an Euler circuit. Write out the Euler circuit if it exists. Justify your answer if it does not exist. [2 marks]

(c) A box contains 6 red and 5 black balls. If 4 balls are selected at random from this box, find the number of ways that

i. Exactly 2 red balls are selected [1 mark]

ii. At least 3 black balls are selected [1.5 marks]

iii. All the selected balls have the same color [1.5 marks]

End of Page

Appendix

Equivalences' Law

Identity laws	**Associative laws**	**Conversion of Implication**
$p \wedge T \Leftrightarrow p$	$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$	$p \rightarrow q \Leftrightarrow \neg p \vee q$
$p \vee F \Leftrightarrow p$	$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	$p \vee q \Leftrightarrow \neg p \rightarrow q$
Domination laws	**Absorption Laws**	**Contrapositive**
$p \vee T \Leftrightarrow T$	$p \vee (p \wedge q) \Leftrightarrow p$	$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$
$p \wedge F \Leftrightarrow F$	$p \wedge (p \vee q) \Leftrightarrow p$	$p \leftrightarrow q \Leftrightarrow \neg q \leftrightarrow \neg p$
Idempotent laws	**Commutative laws**	**Conversion of Equivalence**
$p \vee p \Leftrightarrow p$	$p \vee q \Leftrightarrow q \vee p$	$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
$p \wedge p \Leftrightarrow p$	$p \wedge q \Leftrightarrow q \wedge p$	
Negation laws	**Distributive laws**	**De Morgan's laws**
$p \vee \neg p \Leftrightarrow T$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
$p \wedge \neg p \Leftrightarrow F$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
Double Negation laws	$(p \vee q) \wedge r \Leftrightarrow (p \wedge r) \vee (q \wedge r)$	
$\neg(\neg p) \Leftrightarrow p$	$(p \wedge q) \vee r \Leftrightarrow (p \vee r) \wedge (q \vee r)$	

Rules of Inference

1. Addition: $p \Rightarrow p \vee q$
2. Simplification: $(p \wedge q) \Rightarrow p$
3. Modus Ponens: $[p \wedge (p \rightarrow q)] \Rightarrow q$
4. Modus Tollens: $[\neg q \wedge (p \rightarrow q)] \Rightarrow \neg p$
5. Hypothetical Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$
6. Disjunctive Syllogism: $[(p \vee q) \wedge \neg p] \Rightarrow q$
7. Resolution: $[(p \vee q) \wedge (\neg p \vee r] \Rightarrow (q \vee r)$
8. Conjunction: $p \wedge q \Rightarrow (p \wedge q)$